



Flowmeter Repeatability, Reproducibility, Linearity

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Repeatability is defined as the ability of a flowmeter to reproduce a measurement while being tested under an unchanging set of conditions. For example, testing would be performed by fixing the flowrate and stabilizing ambient conditions and then taking point after point without changing the flow rate setting. It is not to be implied that the indicated flow is correct, but rather that the indication is the same every time.

Reproducibility is in every respect the same as repeatability with one significant exception. During reproducibility testing, the flow rate is changed and then reset between repeats. Reproducibility is therefore a more realistic indication of a flowmeter's ability to reproduce a measurement whenever a predefined set of conditions is recreated.

The terms Reproducibility and Repeatability are some times used interchangeably, but since the former is a more rigorous form of testing, it is some times avoided by marketers determined to present their product in the best light possible.

The results from either form of testing must be analyzed using valid statistical methods. The method of choice in the flow measurement field is **Student's t analysis with 95% C.L.** (Confidence Level). Description is attached in the appendix.

In order to generate a useful indication of the true repeatability/reproducibility characteristics of a flowmeter, a large number of repeats must be taken for each set of test conditions. It is recommended that 30 repeats are taken in order to reduce the value of the t coefficient (uncertainty level).

Obviously 30 repeats at a large number of different sets of conditions would not be practical to perform routinely. Fortunately however, flowmeter performance is usually predicable, consistent and linear. This holds especially true for Turbine flowmeters. As a result, it can easily be shown that repeatability characteristics throughout the entire range of a flowmeter can be evaluated by testing at a limited number of test conditions and even at a single flowrate.

It is recommended that testing is performed at the flowrate of approximately 25-35% of the maximum flowrate of a given flowmeter. In the attached sample results from testing of an ANC 12 flowmeter which has a range of 3.8 to 114 lpm, we have chosen 35 lpm. This selection allows testing in the linear area of the meter and at the same time reasonably close to the bottom end of the flow range where repeatability problems would be more easily detectable.

The attached sample repeatability test results for an ANC 12 demonstrate the above concepts. As can be seen, Student's t analysis shows that determination of repeatability based on 4 repeats is unreliable and can vary by a factor of 3 (0.027 to 0.113% in the example). The same test results used as a group of 30 yielded a reliable and believable measure of **Repeatability of 0.051% with 95% C.L.**

Tests for the same flowmeter yielded a value for Reproducibility of 0.055% again with 95% C.L. As anticipated, it was slightly worse than the repeatability results. One would expect that the difference is the result of the greater variability of the test conditions which is inherent to testing where conditions are intentionally varied and then reset.

But is this really the case?

Linearity is another important characteristic of flowmeter performance. It is a measure of how close the relationship between flow and flowmeter output approximates a linear function. For turbine flowmeters, a constant K-factor (pulses per unit volume) is indication of linearity and the reason why turbine meter performance is plotted as K-factor vs flowrate. In this representation, a horizontal line represents ideal linearity.

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By analyzing the reproducibility test results graphically, it can be clearly seen that the K-factor values increase as flowrate increases. Consequently, some of the 0.055% reproducibility error is due to the non linear nature of the flowmeter performance.

In order to separate the **systematic linearity error** from the **random reproducibility error** the following method has been used as demonstrated in the attached graph of the ANC 12 reproducibility data. A least square line was created with the following formula:

$$y=ax+b \quad \text{or} \quad K=aQ+b \quad \text{Where: } K - \text{K-factor, pul/lit}$$

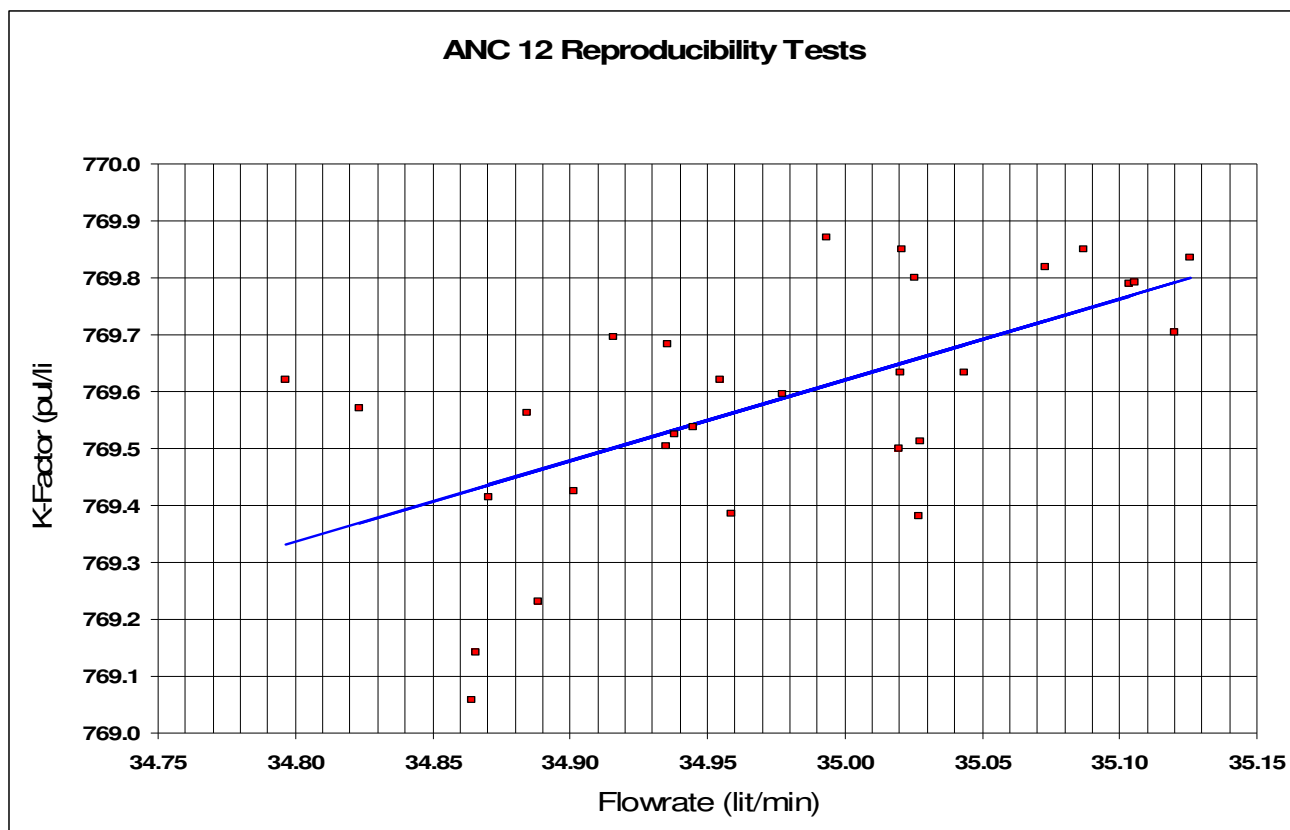
$$Q - \text{Flowrate, lit/min}$$

The constants were determined to be: $a = 1.421$ (slope)
 $b = 719.875$ (intercept)

The deviation of the individual reproducibility points from this line were then calculated and it was then again estimated that the pure **Reproducibility error is 0.043% with 95% C.L.**

This means that the original random reproducibility error calculation actually contained a systematic Linearity error of 0.012% with 95% C.L., or more that a fifth of the total. If not separated by the methods described above, the contribution of linearity to the error calculation is much more significant and in fact becomes predominant when testing is performed in the non-linear region of a flowmeter's range.

This is an important contributing factor to the often mistaken belief that flowmeters exhibit deteriorating repeatability/reproducibility as flowrate decreases.



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ANC 12 Reproducibility test

(flowrate reset between repeats)

Revised data: Deviations calculated from best straight line fit

Freq.	Rate	K-factor	Std. Dev.	Estimated Error		Straight Line Fit	Estim. K-Factor	Delta K-Factor	Estimated Error			
				95% C.L.	99% C.L.				Std. Dev.	95% C.L.	99% C.L.	
Hz	lpm	p/lit	4 consecutive samples running	4 consecutive samples running		Line Fit	p/lit		4 consecutive samples running			
449.130	35.020	769.500				Slope:	769.649	-0.149				
447.568	34.902	769.425				1.421	769.481	-0.056				
450.536	35.120	769.704				Intercept	769.791	-0.087				
449.150	35.027	769.381	0.124	0.051%	0.094%	719.875	769.659	-0.277	0.085	0.035%	0.065%	
450.684	35.126	769.835	0.190	0.078%	0.144%		769.799	0.035	0.114	0.047%	0.086%	
449.236	35.028	769.513	0.174	0.072%	0.132%		769.660	-0.146	0.113	0.047%	0.085%	
448.277	34.959	769.386	0.184	0.076%	0.140%		769.562	-0.176	0.113	0.047%	0.086%	
450.369	35.103	769.789	0.187	0.077%	0.142%		769.767	0.021	0.095	0.039%	0.072%	
448.095	34.938	769.524	0.146	0.061%	0.111%		769.533	-0.008	0.085	0.035%	0.064%	
446.947	34.866	769.142	0.234	0.097%	0.178%		769.430	-0.288	0.126	0.052%	0.096%	
450.196	35.087	769.851	0.279	0.115%	0.212%		769.744	0.106	0.148	0.061%	0.112%	
448.640	34.977	769.595	0.254	0.105%	0.193%		769.588	0.007	0.147	0.061%	0.111%	
447.166	34.871	769.416	0.259	0.107%	0.196%		769.437	-0.021	0.146	0.060%	0.111%	
449.997	35.073	769.819	0.177	0.073%	0.134%		769.724	0.095	0.055	0.023%	0.042%	
449.212	35.020	769.634	0.143	0.059%	0.109%		769.649	-0.015	0.047	0.019%	0.035%	
447.427	34.884	769.564	0.145	0.060%	0.110%		769.456	0.108	0.060	0.025%	0.045%	
450.397	35.105	769.792	0.107	0.044%	0.081%		769.770	0.022	0.051	0.021%	0.039%	
448.366	34.955	769.621	0.085	0.035%	0.064%		769.556	0.064	0.046	0.019%	0.035%	
448.045	34.935	769.505	0.107	0.044%	0.081%		769.528	-0.024	0.049	0.020%	0.037%	
449.377	35.026	769.799	0.123	0.051%	0.094%		769.657	0.142	0.061	0.025%	0.046%	
447.908	34.916	769.696	0.107	0.044%	0.082%		769.501	0.195	0.083	0.034%	0.063%	
447.287	34.888	769.232	0.216	0.089%	0.164%		769.462	-0.230	0.166	0.069%	0.126%	
449.346	35.021	769.850	0.245	0.101%	0.186%		769.650	0.200	0.179	0.074%	0.136%	
448.192	34.945	769.538	0.229	0.095%	0.174%		769.542	-0.005	0.177	0.073%	0.134%	
446.876	34.864	769.058	0.302	0.125%	0.229%		769.427	-0.370	0.217	0.090%	0.165%	
449.510	35.043	769.633	0.290	0.120%	0.220%		769.682	-0.049	0.204	0.084%	0.155%	
446.651	34.823	769.571	0.229	0.095%	0.174%		769.370	0.201	0.205	0.085%	0.155%	
449.005	34.993	769.871	0.296	0.123%	0.225%		769.611	0.260	0.248	0.103%	0.188%	
448.156	34.936	769.684	0.112	0.046%	0.085%		769.529	0.155	0.116	0.048%	0.088%	
446.338	34.797	769.621	0.114	0.047%	0.086%		769.332	0.289	0.052	0.021%	0.039%	
		Mean K	30 samples results				Mean K	30 samples results				
		769.585	0.206	0.055%	0.074%		769.585		0.162	0.043%	0.058%	

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Error Calculation Using Student's t

$$t = (X - \mu) / \sigma = \text{Student's } t$$

Where X = Sample Value

μ = Mean of Population

σ = Standard Deviation of Population

$$s = [\Sigma(X_i - X_m)^2 / v]^{1/2} = \text{Standard Deviation of Sample Population}$$

Where

X_i = Individual Sample Value

X_m = Mean of Sample Population

$v = n - 1$ = Degrees of Freedom

n = Number in Sample

$$\% \text{ error} = \pm t * s$$

Sample t values are given in the table below:

Samples	v=n-1	t with 95% C.L.	t with 99% C.L.
2	1	12.71	63.66
4	3	3.18	5.84
30	29	2.04	2.76
□	□	1.96	2.58

Example:

From attached ANC 12 Reproducibility test data:

Sample population $n = 30$
 $v = 30 - 1 = 29$

Consequently $t = 2.04$ for 95% C.L.

Population Mean $X_m = 769.6$

Sample Std. Dev. $s = 0.206$

$$\text{Error} = t * s = 2.04 * 0.206 = \pm 0.420$$

$$\% \text{ Error} = t * s * 100 / X_m = 2.04 * 0.206 * 100 / 769.6 = \pm 0.055\% \quad \text{with 95\% C.L.}$$